

Symmetry Breaking in Nuclei*

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It is shown that several effects are responsible for deviations of the intensity distributions from the Porter-Thomas law. Among these are genuine symmetry breaking, such as isospin; the nature of the transition operator; truncation of the Hilbert space in shell model calculations and missing transitions.

Random matrix theory (RMT) [1–3] has had a wide application in the description of the statistical properties of eigenvalues and eigenfunctions of complex many-body systems. More general ensembles have also been considered [4], in order to cover situations that depart from the universality classes of RMT. One such class of ensembles is the so-called deformed Gaussian orthogonal ensemble (DGOE) [5–7]. These “intermediate ” ensembles are particularly useful when one wants to study the breaking of a given symmetry in a many-body system such as the atomic nucleus. When discrete symmetries such as isospin, parity and time reversal are violated in a complex many-

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body environment, one relies on a description based on the transition between one GOE into two coupled GOE 's in the first two cases [7,8] and a GOE into a Gaussian Unitary Ensemble (GUE) in the last case.

An important test of the statistical behavior of a complex many-body system is the distribution of the eigenvectors components. If fully chaotic, the system obeys the Porter-Thomas law [9]. This is verified in a variety of cases and using shell-model calculation [10–12]. Some deviations from this distribution were reported which may reflect several of many effects: the limited applicability of the ergodic theorem, some missing strengths and otherwise unknown broken hidden symmetry.

A very important test of the RMT in the context of discrete symmetry breaking has been supplied by Mitchell and collaborators at TUNL where isospin mixing was investigated in the spectrum of ^{26}Al [13]. Here, full experimental knowledge of the discrete spectrum up to 8 Mev of excitation energy is almost complete (about 100 states). Further, owing to the odd-odd character of this isotope (13 protons+13 neutrons) one expects to find both $T=0$ and $T=1$ states even at very low excitation energies. Therefore, ^{26}Al is a very convenient laboratory to study isospin breaking in the nuclear states. Only recently, the study of the transition distribution was made following some suggestions concerning the nature of the manifestation of a discrete symmetry breakdown in the distribution made by Alhassid and Levine [14] and Hussein and Pato [5]. The recent work of TUNL showed a great deviation of the distribution from the pure Porter-Thomas form [15]. A possible cause of this may be a collective state responsible for the T -mixing. No clear explanation, however is given. One reason for this is the

lack of a detailed theoretical study of such deviations. Here we supply a model that allows a better understanding of the effect of symmetry breaking on the transition distribution.

In the following we show that it is possible to introduce a random operator whose matrix elements simulate, in some way, the behavior of observables of complex physical systems. In the construction of this operator, we will be guided by the idea that when a system undergoes a chaos-order transition, a quantity that has a key role in determining the statistical behavior of the matrix elements of an operator, is the expectation value of its commutator with the Hamiltonian. This is implied by the equation

$$(E_l - E_k) \langle E_l | O | E_k \rangle = \langle E_k | [O, H] | E_k \rangle = i\hbar \frac{d}{dt} \langle E_l | O | E_k \rangle \quad (1)$$

where the last equality was obtained using Schrödinger equation and assuming an operator with no explicit dependence on time. Eq.(1) clearly shows that the commutator supplies the connection between the matrix elements of an observable and its behavior as a function of time.

To see how this is reflected in the statistical distribution, we consider an observable O which becomes a conserved quantity in the regular regime. The distribution of the matrix elements of O will undergo a transition from the Porter-Thomas law, in the chaotic regime, to a distribution corresponding to the singular $\langle E_l | O | E_k \rangle \propto \delta_{kl}$, in the regular regime, since the last term in (1) is zero in this latter case. When considering the question of violation of a discrete symmetry in a complex system one resorts to a description involving the transition $2\text{GOE} \rightarrow 1\text{GOE}$, where it is assumed that in the

first limit (2GOE) one has a conserved quantum number that labels two sets of levels (e.g., parity or T=0 and T=1 states). The opposite limit (1GOE) involves the complete breakdown of the symmetry. Then, in general, for some weak symmetry violation, one has two coupled GOE's. The operator O may have non-zero matrix element for the transitions belonging to the two different GOE blocks of the Hamiltonian matrix. This causes a deviation of the distribution from the P-T law. However, even if the symmetry is not broken, pieces of the operator O may also couple these states and will also result in deviation from the P-T law. Here is the dilemma which we will address in the following: how to distinguish genuine symmetry breaking effects from false ones related to the nature of O through the study of the deviations from the P-T law. Of course, missing transitions (not counted in the analysis) also result in a significant deviation.

To define the ensembles of random matrices we are going to work with, we follow the construction based on the Maximum Entropy Principle [5], that leads to a random Hamiltonian which can be cast into the form

$$H = H_0 + \lambda H_1, \quad (2)$$

where λ ($0 \leq \lambda \leq 1$) is the parameter that controls the chaoticity of the ensemble. For $\lambda = 1$, $H = H^{GOE}$, the fully chaotic situation, whereas for $\lambda = 0$, we have a different situation defined by H_0 (e.g. two uncoupled GOE's). Since we are specifically interested in the transition from GOE to two uncoupled GOE's, we write [5,6]

$$H_0 = PH^{GOE}P + QH^{GOE}Q \quad (3)$$

and

$$H_1 = PH^{GOE}Q + QH^{GOE}P \quad (4)$$

where $P = \sum_{i=1}^M P_i$, $Q = 1 - P$ and $P_i = |i\rangle\langle i|$, $i = 1, \dots, N$ are projection operators. Therefore H_0 is a two blocks diagonal matrix of dimensions M and $N - M$ and each block is by construction a GOE random matrix. It is easily verified that $H = H^{GOE}$ for $\lambda = 1$.

There are two main classes of operators to be considered. The first class is the set of symmetry conserving, O_C operators which in the limit of conserved symmetry, $\lambda = 0$, i.e., two GOE's in our model, have elements only inside the blocks while, the other class is that of the symmetry breaking O_B ones which in the same limit have elements only between the blocks. Starting with the O_C ones, they can be simulated in the model by an operator of the form

$$O_C = \sum_{i=0}^N P_i H^{GOE} P_i. \quad (5)$$

To see how it acts, it is convenient to separate the sum in Eq. (5) into two parts in which the first M terms define the operator O_{CP} and the other $N - M$ define the operator O_{CQ} , by construction $O_C = O_{CP} + O_{CQ}$. The commutator with the Hamiltonian can then be written as

$$[O_{CP}, PHP] + [O_{CQ}, QHQ] + \lambda (O_{CP}PHQ + O_{CQ}QHP - QHPO_{CP} - PHQO_{CQ}). \quad (6)$$

The first and the second terms are block diagonal while those multiplied by λ are block

anti-diagonal. In the limit $\lambda \rightarrow 0$, the eigenstates become localized in the blocks and only the transitions between states inside the blocks PHP and QHQ survive.

In the case of symmetry breaking operators O_B , they can be simulated simply by H_1 , i.e., $O_B = H_1$. In fact, its commutator with the Hamiltonian is

$$[H, O_B] = PHPHQ + QHQHP - PHQHQ - QHPHP \quad (7)$$

which is block anti-diagonal. So, as consequence, when $\lambda \rightarrow 0$ this operator induces transitions only between the blocks.

The above properties of these operators make them very convenient to study the situation when a selection rule becomes operative in a transition towards two coupled GOE's, a scenario appropriate to investigate discrete symmetry violation, e.g., isospin, in nuclei. In order to make our model more flexible we are going to study distributions of matrix elements of the generic operator

$$O = (1 - q) O_C + q O_B \quad (8)$$

where q varies between 0 and 1. With this more generic form the model depends on two parameters, the parameter λ of the Hamiltonian which may be fixed by the fitting the eigenvalues distribution and the parameter q that selects the operator.

Following the standard procedure, we first construct, with the operator O , the normalized vector

$$| \alpha_k \rangle = O | E_k \rangle \quad (9)$$

where $| E_k \rangle$ with $k = 1, \dots, N$ is an eigenvector of the Hamiltonian. From these N vectors we define the matrix elements

$$T_{kl} = \langle E_l | \alpha_k \rangle \quad (10)$$

which are the quantities to be statistically analyzed. It is convenient to work with $|T_{kl}|^2$, and perform a local average that extracts secular variation with the energies. Thus we introduce the quantities

$$y_{kl} = \frac{|T_{kl}|^2}{\langle |T_{kl}|^2 \rangle} \quad (11)$$

where the average is done by using a Gaussian filter of variance equal to 2 and, as it has become standard in the analysis of these quantities, we histogram their logarithm.

In Fig. 1, it is shown the numerical simulations performed with the symmetry conserving operator, O_C , for four values of the parameter λ . The striking characteristic of these plots is the strange dependence of these statistics on the chaoticity parameter. The evolution of the distribution with λ is clearly seen to be P-T \rightarrow non P-T \rightarrow P-T. Thus, great care must be taken when confronting deviation from P-T from genuine symmetry breaking. Very strong symmetry breaking could lead to a P-T distribution.

In Fig. 2, the results are shown for the distribution of the matrix element of the general operator O . The chaoticity parameter was fixed at value $\lambda = 0.032$ which is so fixed to account for the isospin breaking seen in the eigenvalue distribution of Ref. [13], and the parameter q , that measures the “symmetry deformation” of O , is varied. At the above value of λ , we expect to be near the case in which the two GOE’s are nearly decoupled. We see from the figure that the distributions are strongly dependent on the parameter q . Thus, very weak symmetry violation in the system may result in a strong deviation of the transition strength distribution from a P-T one owing to the

nature of the transition operator.

One important question from the experimental side is the effect of missing strengths in the data analysis. The experimental apparatus may have a minimum below which strengths are not detected or, else, it may not be able to resolve a relatively weak strength staying close to a strong one. The fact that the data is not complete may result in a distortion of the final distribution. This may be the case, for example, of the measured distribution of Ref. [15], in which besides the shift to the left, in qualitative agreement with our theoretical distributions, the data exhibits a peak higher than the Porter-Thomas distribution. In order to check this explanation, we have performed a simulation of the effect in our model, by eliminating, from the distribution, matrix elements below some threshold value. The result obtained is shown as the histogram in Fig 3 together with the P-T distribution. Indeed, it is seen in the distorted distribution the same qualitative trend as the data analysis of Ref. [15], namely, an enhancement of the maximum which is shifted to the left. Therefore, missing transitions *also* lead to significant deviation from the P-T law.

It is interesting to observe at this point that all the deviations discussed above can be represented most adequately by the sum of two χ^2 -distributions as was suggested earlier [6].

Before ending, it is of interest to briefly discuss some recent result of extensive shell model calculation of the intensity distribution. Hamoudi, Nazmitdinov and Alhassid [16] made statistical fluctuation analysis of electromagnetic transition intensities in A=60 nuclei using the shell model code OXBASH. They specifically looked at E2 and

M1 transitions among $T=0,1$ states. They found that $B(E2)$ distributions are well described by the P-T law, independently of the value of $T_z = \frac{N-Z}{2}$. However, they point out that $B(M1)$ distributions depend strongly on T_z : $T_z = 1$ nuclei obey the P-T law while $T_z = 0$ show significant deviation from the P-T prediction.

In light of our discussion and considerations above, the fact that very small violation of T is expected we conjecture that the deviation in Ref. [16] is a “false ” symmetry breaking violation effect that arises from the truncation of the basis in the calculation. Further study is certainly needed to better understand the phenomenon.

In conclusion, we have investigated in this letter possible causes for the strength intensity distributions to deviate from the Porter-Thomas law. We can trace the deviations to: genuine discrete symmetry breaking, such as isospin; the nature of the transition operator; truncation of the Hilbert space (which requires dealing with effective transition operators that may contain “false ” symmetry violation terms) and “missing transitions”. An analysis of the data of Ref. [15] using our model is underway.

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Figure Captions:

Fig. 1 Four histograms of the logarithm of the matrix elements distributions of the random operator O_C for the transition $\text{GOE} \rightarrow 2\text{GOE}$'s, for the indicated values of the parameter λ . The calculations were done with matrices of dimension $N = 150$ and block sizes $M_1 = 80$ and $M_2 = 70$. The dashed line corresponds to P-T, the dotted line to the fit with one χ^2 - distribution and the solid one to a sum of two χ^2 - distributions. See text for details.

Fig. 2 Four histograms of the logarithm of the matrix elements distributions of the generic random operator O for the case of the transition $\text{GOE} \rightarrow 2\text{GOE}$'s, for the indicated values of the parameter q . The calculations were done with matrices of dimension $N = 150$ and block sizes $M_1 = 80$ and $M_2 = 70$. The lines are as in Fig. 1.

Fig. 3 The effect of the missing transitions in the P-T distribution. The calculations were done with matrices of dimension $N = 100$. See text for details.





